

# Losses in three-phase transformers at load balancing

## Pérdidas en transformadores trifásicos en equilibrio de carga

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#### ABSTRACT:

A mathematical model of a transformer that takes into account the type of magnetic circuit, as well as losses in its constructive elements from zero sequence currents is proposed. The nature of the dependence of the transformer losses on the difference in the conductivities of the loads of individual phases is shown with the total conductivity unchanged. Based on the results of computer simulation, the relationship between the total power losses and the unbalance coefficients of the zero and negative sequence and coefficient of asymmetry is determined. An estimate is given for reducing additional power losses in transformers when using a three-phase load equalization device. The appearance of the experimental sample of the load balancing unit is presented. key

**Keywords:** loss of electric power, three-phase transformer, unbalanced load, load balancing, power quality.

#### RESUMEN:

Se propone un modelo matemático de un transformador que tiene en cuenta el tipo de circuito magnético, así como las pérdidas en sus elementos constructivos a partir de corrientes de secuencia cero. La naturaleza de la dependencia de las pérdidas del transformador en la diferencia en las conductividades de las cargas de fases individuales se muestra con la conductividad total sin cambios. Con base en los resultados de la simulación por computadora, se determina la relación entre las pérdidas totales de potencia y los coeficientes de desequilibrio de la secuencia cero y negativa y el coeficiente de asimetría. Se proporciona una estimación para reducir pérdidas de potencia adicionales en transformadores cuando se utiliza un dispositivo de equalización de carga trifásica. Se presenta la apariencia de la muestra experimental de la unidad de equilibrio de carga.

**Palabras clave:** pérdida de energía eléctrica, transformador trifásico, carga desequilibrada, equilibrio de carga, calidad de la energía.

## 1. Introduction

Unbalanced load conditions are common phenomenon in 0.4 kV electric mains. A systematic load inequality by phases can occur even in qualitatively designed electrical networks when

they grow. Unbalance of currents is the cause of voltage deviations at points of load common coupling. In some cases, such deviations exceed the values adopted in the standard [4], which can lead to negative consequences for the electrical equipment of end users. [11]

Unbalanced load are a source of losses in the elements of electrical mains. The relevance of this problem is confirmed by large number of papers. For example, in work [8, 10] authors present algorithms and computer programs for determining the energy losses in the elements of electrical mains caused by decrease in the quality of electricity. In work [10] Miroshnik A.A. estimates the degree of energy losses in case of unbalancing load. In [5, 6, 7] Ded A.V. et al. describe the influence of non-symmetry on power losses in various elements of electrical mains.

There are papers dedicated to various ways for reduce the asymmetry of currents and voltages. For example, in [12, 13, 14, 15] is considered the load balancing device redistributing the load between the phases of the electrical main and the algorithms of its operation.

There are known papers devoted to the modeling of transformers and the determination of additional losses in them with asymmetry. The paper [16] gives a mathematical model in time domain of a three-phase transformer with a common magnetic core taking into account the loss of steel. The work [3] is devoted to modeling of a transformer with a three-rod magnetic core in frequency domain. The model proposed by the authors makes it possible to calculate the magnitudes and angles of currents and voltages in the windings. A number of other works are devoted to the problem of minimizing losses in power transformers, for example, [9, 17]. The losses in the constructive elements of the transformer induced by zero sequence magnetic fluxes are a significant component of the losses in the presence of zero sequence currents. Accounting for this factor in the modeling of transformers, along with the development and research of methods for reducing losses, seems to be an urgent task.

The purpose of the paper is to investigate the transformer losses at unbalanced loads, based on the mathematical model of the transformer, as well as in evaluating the effect of the load balancing device on losses in transformers. Scientific novelty of the work is to develop a mathematical model of a transformer in time domain, taking into account the type of magnetic core and the losses in constructive elements from zero sequence magnetic fluxes, as well as to evaluate the effect of the load balancing device on reducing additional losses in transformers.

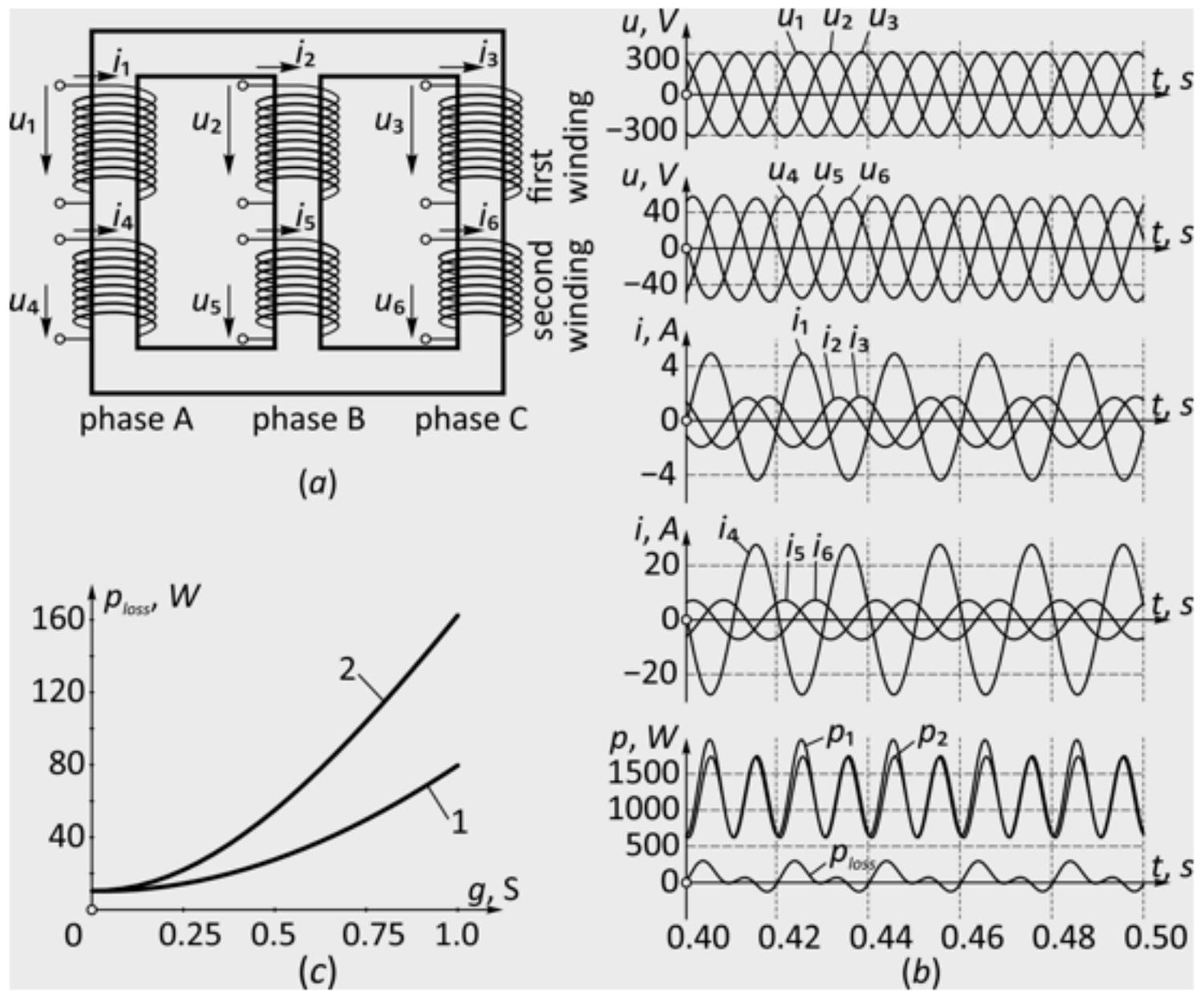
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## 2. Mathematical model

A three-phase two-winding transformer consisting of 6 inductively coupled coils, 2 for each phase, Fig. 1 (a). Each coil in the mathematical model is represented as an active-inductive loop. The contours have a certain inductive coupling between themselves, additionally depending on the type of magnetic circuit. The losses in the constructive metallic elements of the transformer resulting from the circuiting of zero sequence magnetic fluxes through them is made in the model by adding an additional short-circuited contour (the parameters of which are denoted by the index  $sc$ ), which has a low inductive coupling with the coil circuits. The saturation of the magnetic core in the model was not taken into account.

### Fig. 1

(a) The designations of the windings of a three-phase transformer and one of inductively coupled circuits. (b) Oscillogram of the main electrical quantities of the transformer. (c) Dependence of the total power losses  $p_{loss}$  on the total conductance of the load  $g$ : 1 – with a symmetrical load; 2 – with an asymmetrical load.



The approach described in [2] was used in developing the mathematical model. The mathematical description of the transformer model was considered in the form of a first-order ODE system in matrix form.

$$U = R \cdot I + L \cdot di/dt, \quad (1)$$

where  $U = [u_1, u_2, \dots, u_6, u_{sc}]^T$  and  $I = [i_1, i_2, \dots, i_6, i_{sc}]^T$  is the vector of the instantaneous voltages and currents of the coils;  $R = \text{diag}[R_1, R_2, \dots, R_7, R_{sc}]$  is the matrix of coil resistance;  $L$  is the matrix of inductances, the elements of the main diagonal of which are the self-inductances of the coils, the other elements are mutual inductances. The matrix of inductances depends on the type of magnetic circuit. Thus, in a three-rod transformer, all circuits have an inductive coupling, and the matrix of inductances has the form:

$$L_{3rod} = \begin{bmatrix} L_{11} & L_{12} & L_{1sc} \\ L_{21} & L_{22} & L_{2sc} \\ L_{sc1} & L_{sc2} & L_{sc.sc} \end{bmatrix}$$

In a group transformer, each phase can be considered independently, in this case the matrix of inductances will have the form:

$$L_a = \begin{bmatrix} L_{11} & L_{14} \\ L_{41} & L_{44} \end{bmatrix}, L_b = \begin{bmatrix} L_{22} & L_{25} \\ L_{52} & L_{55} \end{bmatrix}, L_c = \begin{bmatrix} L_{33} & L_{36} \\ L_{63} & L_{66} \end{bmatrix}.$$

The model allows to investigate electromagnetic processes in the transformer for arbitrary input actions. In the work, the case of the influence of only sinusoidal supply voltages was investigated, for a precise description of electromagnetic processes, the transformer model was considered in the time domain for instantaneous values. The input voltages were given by:

$$u_1 = U_{m1} \sin(\omega t);$$

$$u_2 = U_{m2} \sin(\omega t - 2\pi/3);$$

$$u_3 = U_{m3} \sin(\omega t + 2\pi/3);$$

$$u_4 = u_5 = u_6 = u_{sc} = 0.$$

The active loads were considered. Resistance of coils

$$R_1 = R_2 = R_3 = R_{s1},$$

$$R_4 = R_{s2} + R_a,$$

$$R_5 = R_{s2} + R_b,$$

$$R_6 = R_{s2} + R_c,$$

where  $R_{s1}$  and  $R_{s2}$  are the self resistances of the primary and secondary windings;  $R_a, R_b, R_c$  – resistance of phase loads.

The total power losses in the transformer  $p_{loss}$  were calculated as the averaged value of the difference of instantaneous active power [1] from the primary  $p_1$  and secondary  $p_2$  windings in the steady state:

$$p_{loss} = p_1 - p_2;$$

$$p_1 = u_1 i_1 + u_2 i_2 + u_3 i_3 + R_{sc} i_{sc}^2;$$

$$p_2 = R_a i_4^2 + R_b i_5^2 + R_c i_6^2.$$

### 3. Results of modeling

The calculation of electromagnetic processes in the transformer was carried out for direct connection to a symmetrical three-phase system of voltages at different loads  $R_a, R_b, R_c$ . The nature of the load is chosen to be purely active due to its wide distribution in distribution networks of 0.4 kV. The connection scheme the Y/Y0 windings. The ODE system (1) was solved numerically with respect to the column vector of the current  $I$  with zero initial conditions. To solve the system of equations and automate a series of numerical experiments, we used the

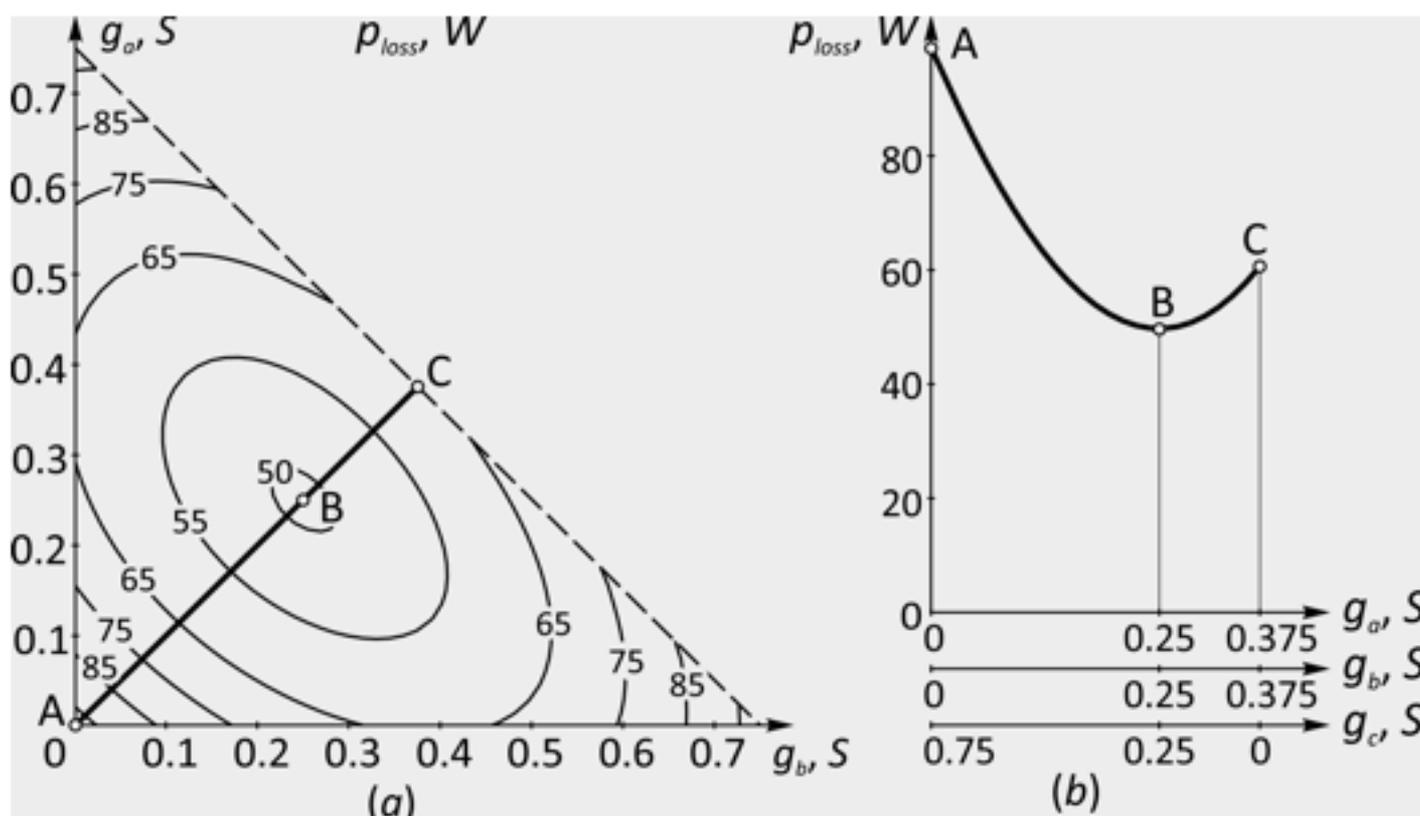
program created by the authors written in Python, which implements the Gauss-Legendre method of 6 order of accuracy. The modeling conditions imply the possibility of changing the load of the transformer within a wide range – from no-load to short circuit – realized in the model due to the time variation of the matrices  $R$  and  $L$ . Applicable method refers to the group of implicit Runge-Kutta methods and is chosen because of the higher stability of the solutions in comparison with explicit methods.

The transformer TC3M-1,6 with nominal phase voltages of 220/42 V, whose parameters are determined experimentally, was considered as a simulation object. Fig. 1 (b) shows the oscillograms of the main electrical quantities of the transformer, as a result of simulation. The oscillograms correspond to a time interval of 0.4–0.5 s after is direct connecting of the transformer to the electric main with an unbalanced load  $R_a = 2 \text{ Ohm}$ ,  $R_b = 8 \text{ Ohm}$ ,  $R_c = 8 \text{ Ohm}$ . To exclude the effect of transients on simulation results, the parameters with new input data were determined in steady-state mode on time interval of 2–3 seconds after the start of simulation.

As the value characterizing the total load of the transformer, the total conductivity of the three phases of the load  $g = g_a + g_b + g_c$  is chosen. It should be noted that the conductivity of the individual phases load is practically proportional to the currents in them in the operating range of the transformer loads. In order to characterize the unbalance, the unbalance coefficient of the currents in zero and negative sequences  $K_0$  and  $K_2$  was used, as well as coefficient of asymmetry  $kasymm = 3 \cdot (I_{a2}^2 + I_{b2}^2 + I_{c2}^2) / (I_a + I_b + I_c)^2$ . This value was used by the author of [4] to estimate additional losses in the elements of the power supply system. For a purely active load in the case under consideration the unbalance coefficients  $K_0$  and  $K_2$  are equal. In this case the relation between the coefficients of asymmetry  $kasymm$  and  $K_0, K_2$  it is characterized by the expression  $kasymm = 2 \cdot K_0^2 + 1$ .

It is known that the total losses of a three-phase transformer include a conditionally constant component associated with magnetic losses and a component proportional to the square of the load amount  $g^2$ . If  $g$  is unchanged, the total power losses of  $p_{loss}$  differ, depending on the degree of unbalance. This difference is shown in Fig. 1 (c), in which curves 1 and 2 correspond to the balanced ( $g_a = g_b = g_c = g / 3$ ) and extremely unbalanced mode ( $g_a = g_b = 0, g_c = g$ ) by the same total conductivity  $g$ .

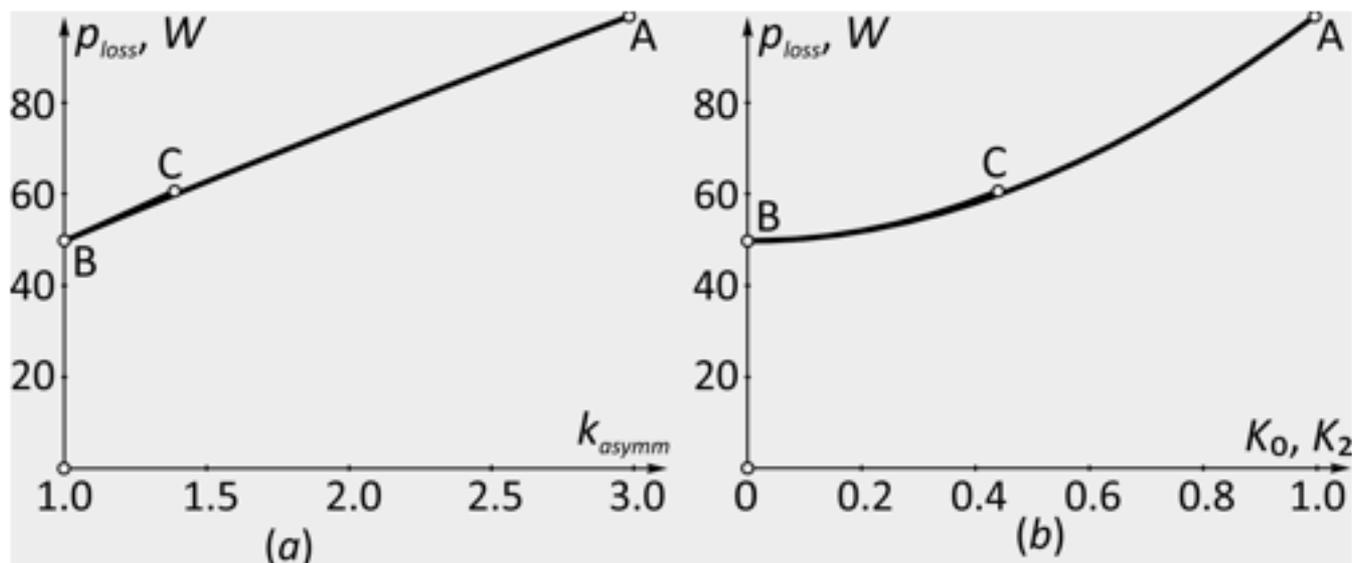
**Fig. 2 (a)** Dependence of the total power loss  $p_{loss}$  by various phase conductivities of loads, but the unchanged total conductivity  $g = 0.75 \text{ S}$  **(b)** Dependence of total losses  $p_{loss}$  when the loads of two phases vary in the range from 0 to  $g/3$ , and the third phase - from  $g$  to  $g/3$ .



The purpose of the investigation was also determination of the character of the dependence of the transformer losses on the degree of unbalance for a given total conductivity  $g = \text{const}$ . For this purpose, the total losses of the transformer were calculated for all practically admissible distributions of phase conductivities  $g_a$ ,  $g_b$  and  $g_c$ . By  $g = \text{const}$  in the coordinates  $g_a$ ,  $g_b$  and  $g_c$  the set of conductivity values form a plane in the first octant.

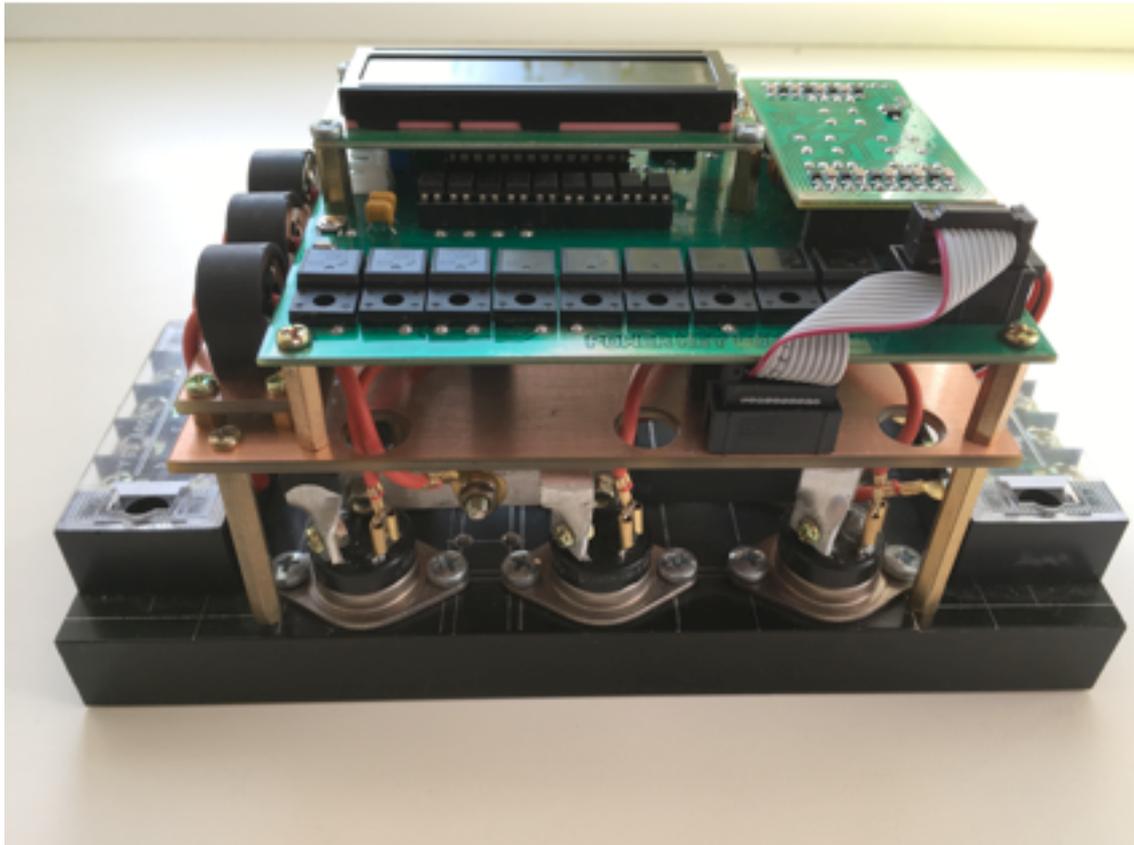
Fig. 2 (a) shows the dependence of the total power loss  $p_{\text{loss}}$  by different phase conductivities of loads, but by the unchanged total conductivity  $g = 0.75 \text{ S}$  in the form of a contour plot. The graph is based on the results of computer simulation of the transformer with various combinations of phase conductivities of loads. Under the conditions under consideration, the graph is symmetrical with respect to the point with phase conductivities of the load  $g_a = g_b = g_c = g / 3 = 0.25 \text{ S}$ . At a given point total losses  $p_{\text{loss}}$  are  $50 \text{ W}$ . With increasing unbalance, the total losses increase and reach a maximum at the corners of the contour plot, where the conductivities of the two phases are zero, and the conductivity of the third phase is  $g$ . Fig. 2 (b) shows the total losses when the loads of the two phases vary in the range from 0 to  $g / 3$ , and the third phase are from  $g$  to  $g/3$ , which corresponds to the ABC line in the contour plot in Fig. 2 (a). This is one of the possible ways of changing the unbalance coefficients of currents in the reverse and zero sequences  $K_0$  and  $K_2$  in the range from 1 to 0, and the load coefficients of asymmetry  $k_{\text{asymm}}$  is from 3 to 1. Fig. 3 (a, b) shows the dependence of the total power loss  $p_{\text{loss}}$  on the coefficients of asymmetry  $k_{\text{asymm}}$ , as well as on the coefficients of asymmetry of the currents in negative and zero sequences  $K_0$  and  $K_2$ . The graphs show that the total power losses of  $p_{\text{loss}}$  are proportional to  $K_0^2$ ,  $K_2^2$  and  $k_{\text{asymm}}$ .

**Fig. 3 (a)** Dependence of the total losses of the power  $p_{\text{loss}}$  on the load coefficients of asymmetry. **(b)** Dependence of the total losses of the power  $p_{\text{loss}}$  on the unbalance coefficients of the currents on the negative and zero sequence  $K_0$  and  $K_2$ .



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**Fig. 4.** Appearance of the experimental sample of the load balancing unit.



An experimental sample of the LBU for low-voltage distribution networks is shown in fig. 4. The power section of the device is made on 9 triacs mounted on the radiator. The radiator serves as the main structural element of the experimental sample of LBU, and also provides heat removal from the power keys. The triac is controlled by the ATmega 8A microcontroller. To display the current position of the power keys on the controller board, an LCD display is provided. To connect the device to the electric mains and to the load, there are two terminal blocks.

Also in the device there is a set of current transformers for measuring the currents of the outgoing load line connected to the device, as well as a connector for connecting to the phases of the electrical network and a neutral wire for measuring phase voltages by the control system. The interaction of the power key controller and the control system is provided via the UART interface wired or wirelessly by uses the appropriate adapters.

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## 4. Conclusions

1. A mathematical model of a transformer is proposed that takes into account the type of magnetic circuit, as well as losses in constructive elements from the component of the zero sequence currents.
2. The nature of the dependence of the losses of the transformer on the degree of load asymmetry for a given total conductivity is established.
3. Based on the results of computer simulation, the relationship between the total power losses with the active load profile with zero and negative sequence unbalance currents and the load non-uniformity coefficient is shown.
4. It is shown that the load balancing unit allows, in the optimal case, to reduce additional power losses in the transformer by an average of 48% compared to this value without using the device.

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